

Emergence of Continuous Signals as Shared Symbols Through Emergent Communication

Issei Saito

The University of Electro-Communications
Tokyo, Japan
The State University of New York
at Binghamton
Binghamton, NY
i_saito@radish.ee.uec.ac.jp

Tadahiro Taniguchi

Ritsumeikan University
Kusatsu, Shiga
taniguchi@em.ci.ritsumeai.ac.jp

Tomoaki Nakamura

The University of Electro-Communications
Tokyo, Japan
nakamura@radish.ee.uec.ac.jp

Yohei Hayamizu

The State University of New York
at Binghamton
Binghamton, NY
yhayami1@binghamton.edu

Akira Taniguchi

Ritsumeikan University
Kusatsu, Shiga
a.taniguchi@em.ci.ritsumeai.ac.jp

Shiqi Zhang

The State University of New York
at Binghamton
Binghamton, NY
zhangs@binghamton.edu

Abstract—Humans create shared symbols through interactions with others and the environment, enabling effective communication within a specific community. This process, known as emergent communication, facilitates the exchange of intentions and meanings. Furthermore, humans combine phonemes, the smallest units of sounds, to create a myriad of continuous symbols, a property called compositionality. The generation and recognition capabilities of these speech signals are acquired through interactions with others. Although existing studies have promoted the emergence of discrete symbols representing observed objects through two agents, the emergent creation of continuous signals with compositionality is yet to be studied. This study proposes a novel probabilistic generative model that integrates the Metropolis-Hastings naming game (MHNG) and Gaussian process hidden semi-Markov model (GP-HSMM), allowing the emergence of continuous symbols. MHNG allows symbol emergence through communication between independent individuals without directly observing the internal states of others. Moreover, GP-HSMM supports the unsupervised segmentation of continuous signals and acquires continuous signals with compositionality as shared symbols. We conducted experiments using the upper and lower portions of MNIST as the observed images to evaluate the efficacy of our proposed approach. The results demonstrate that the agents successfully generate and recognize shared continuous signals representing MNIST digits despite possessing different internal representations.

Index Terms—Metropolis-Hastings naming game, GP-HSMM, emergent communication, probabilistic generative model

I. INTRODUCTION

Humans can create shared symbols through interactions with others and the environment to communicate intentions in a specific community. This process of creating emergent shared symbols is called emergent communication. Humans use continuous speech signals, combining phonemes, the

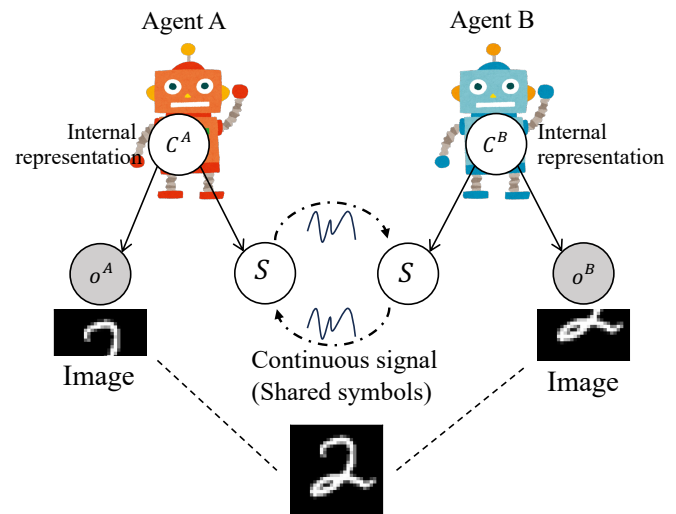


Fig. 1: Overview of the proposed method and task.

smallest units of speech sound, as symbols to create numerous continuous symbols, referred to as compositionality. The speech signals' generation and recognition capabilities are learned through interactions with others.

In recent years, research has focused on emergent communication using deep neural networks [1]–[5]. Most studies utilized unidirectional communication from the sender to the receiver, focusing on tasks such as referential games [6]–[8], where agents use reinforcement learning to choose appropriate targets through communication. Additionally, deep reinforcement learning with communication channels [9], [10] can be considered as agents creating emergent shared symbols. These

This work was supported by JST Moonshot R&D Grant No. JPMJMS2011.

approaches involve interconnected agents sharing information through a network during learning and backpropagating the error information calculated from the internal states and observations of others to each network. This enables each agent to learn actions by considering the states of others. However, humans are independent individuals and cannot directly utilize the information calculated from the internal states of others. Considering emergent communication among independent individuals, such a setup in which an agent utilizes the information of the states of others is considered unnatural. Regarding this matter, Taniguchi et al. proposed the Collective predictive coding hypothesis to elucidate the emergence of symbols from a neuroscience perspective [11]. This hypothesis accentuates a distributed and predictive coding-driven process in which symbols emerge through the individual agents' representation learning and social interactions. Based on this hypothesis, Taniguchi et al. proposed the Metropolis-Hastings naming game (MHNG) method, in which two agents create shared symbols without directly observing others' internal states and acquiring their meanings [12], [13]. Further, Okumura et al. demonstrated that human's behavior in a certain symbol emergence task was consistent with the MHNG theory in their study [14]. These studies promoted the emergence of discrete symbols representing observed objects through two agents; however, it has not yet been extended to the emergent creation of continuous signals with compositionality.

Therefore, in this study, we propose a probabilistic generative model that combines MHNG and Gaussian process hidden semi-Markov model (GP-HSMM) [15], [16] to represent continuous signals. Fig. 1 provides an overview of the targeted task and the proposed approach. Two agents observe the same object; however, it is natural for their observations not to coincide. To emulate this scenario, we use the upper and lower portions of MNIST as the observations of each agent. The internal representations c^* of each agent and the shared continuous signal S^* representing the observation were inferred through communication via the generated continuous signal.

II. PRELIMINARY

A. Metropolis-Hastings naming game

Our proposed algorithm was inspired by MHNG and learns shared continuous signals with compositionality. As a preliminary step, we first explain MHNG, using the case of the inference of symbol s , where s is a discrete scalar, from the observations of both agents in the graphical model in Fig. 2(a). In the MH algorithm, s^* is sampled from the proposal distribution and accepted or rejected based on the acceptance rate, generating samples for the target distribution. Therefore, the target distribution is given as follows:

$$P(s) = p(s|o^A, o^B) \quad (1)$$

$$\approx Z p(s|o^A) p(s|o^B), \quad (2)$$

where Z is a regularization term. The product-of-experts (PoE) approximation is used in this equation-transformation. Next,

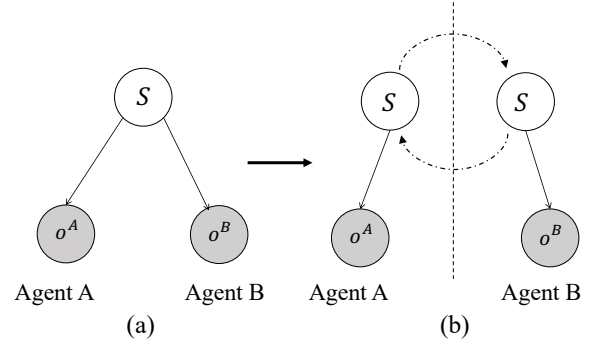


Fig. 2: Model decomposition based on Metropolis-Hastings naming game.

when Agent A proposes a symbol, the proposal distribution is given as follows:

$$Q(s^*) = p(s|o^A) \quad (3)$$

Next, we determine whether to accept or reject the proposed s^* according to the acceptance rate based on the MH algorithm, as follows:

$$r = \frac{P(s^*)Q(s)}{P(s)Q(s^*)} \quad (4)$$

$$= \frac{p(s|o^A)p(s^*|o^A, o^B)}{p(s^*|o^A)p(s^*|o^A, o^B)} \quad (5)$$

$$= \frac{p(s^*|o^B)}{p(s|o^B)} \quad (6)$$

Equation (4) can be transformed into Equation (6) using Equations (2) and (3). The acceptance rate of s^* proposed by Agent A is calculated solely based on the parameters of Agent B. That is, it is possible to make acceptance/rejection decisions in the MH method without knowing the internal state of the other agent.

These steps are repeated, alternating roles, until convergence, to infer the optimal symbol s . Therefore, using MHNG, the graphical model can be decomposed into the model in Fig. 2(b), where two agents are separated, and shared symbols can be inferred by communication: proposal and accepting/rejecting.

B. GP-HSMM

We introduced GP-HSMM to generate and recognize continuous signals. GP-HSMM allows unsupervised segmentation that divides continuous signals into minimum unit signals, called segments, and classifies them based on their similarities.

1) *Unsupervised segmentation based on GP-HSMM*: Fig. 3 shows a graphical model of the GP-HSMM. A continuous signal S is generated through the following generative process. First, $\hat{c}_j (j = 1, 2, \dots)$ represents the signal class determined by the previous signal class $\hat{c}_{j-1}$ and the transition probability π_c :

$$\hat{c}_j \sim P(\hat{c}|\hat{c}_{j-1}, \pi_c). \quad (7)$$

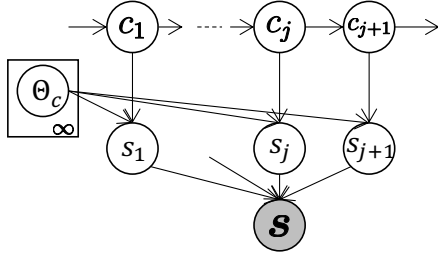


Fig. 3: Graphical model of the Gaussian process hidden semi-Markov model

Segment s_j corresponding to signal class $\hat{c}_j$ is generated from a Gaussian process with parameters $\mathbf{X}_{\hat{c}_j}$.

$$s_j \sim \mathcal{GP}(s|\mathbf{X}_{\hat{c}_j}) \quad (8)$$

The continuous signal S is generated by concatenating the segments s_j .

Inferring the parameters of this model allows a continuous signal to be divided into shorter unit signals s_j . Then, the classes of these segmented unit signals $\hat{c}_j$ can be estimated unsupervised. In this paper, this process is represented as follows:

$$\hat{c}_1, \hat{c}_2, \dots, \hat{c}_L \leftarrow \text{GP-HSMM}(S|\Theta), \quad (9)$$

where Θ represents the parameters of GP-HSMM. For details on the parameter inference, refer to [15], [16].

2) *Generation of continuous signals based on GP:* In GP-HSMM, given the pairs $(\hat{t}_e, \mathbf{S}_{\hat{e}})$ of signal s_t at time step t classified into class $\hat{c}$, the predictive distribution of $\hat{s}_t$ at time step t follows a Gaussian distribution:

$$p(\hat{s}_t|t, \mathbf{S}_{\hat{e}}, \hat{t}_e) \propto \mathcal{N}(\mathbf{k}^T \mathbf{C}^{-1} \mathbf{S}_{\hat{e}}, k(\hat{t}, \hat{t}) - \mathbf{k}^T \mathbf{C}^{-1} \mathbf{k}) \quad (10)$$

$\mathbf{C}$ is a matrix whose p rows and q columns are given by

$$C(t_p, t_q) = k(t_p, t_q) + \phi^{-1} \delta_{pq}, \quad (11)$$

ϕ is a hyperparameter representing the noise in the observed values, and $\mathbf{k}$ is a vector with $k(t_p, \hat{t})$ as its p -th element. In this study, the following equation is used as the kernel function:

$$k(t_p, t_q) = \theta_0 \exp(-\frac{1}{2} \theta_1 \|t_p - t_q\|^2) + \theta_2 + \theta_3 t_p t_q, \quad (12)$$

where θ_* represents the kernel hyperparameters. Therefore, a continuous signal of signal class $\hat{c}$ is generated from the following distribution:

$$\mathcal{GP}(s|\Theta, \hat{c}) = \prod_t p(\hat{s}_t|t, \mathbf{S}_{\hat{e}}, \hat{t}_e), \quad (13)$$

where Θ represents the GP-HSMM parameters.

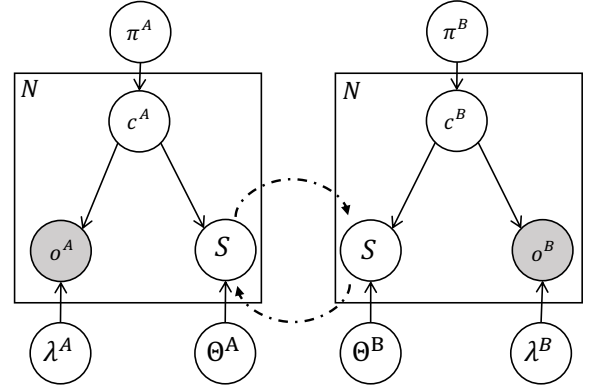


Fig. 4: Graphical model of the proposed method.

III. PROPOSED METHOD

Fig. 4 shows a graphical representation of the proposed method. S represents the continuous signal representing the symbols emerging between agents, and o is the observed information. Class c^* is an internal representation of each agent and captures the co-occurrence of the observations and continuous signals. This model allows the generation of a continuous signal from a class or the inference of a class from a continuous signal.

A. Generative process

Observation o^A and continuous signal S are assumed to be generated as follows: First, the class is generated from a Dirichlet-multinomial distribution with parameter π^* :

$$c^A \sim p(c^A|\pi^A). \quad (14)$$

The observation is generated from a Dirichlet-multinomial distribution with parameter λ^A :

$$o^A \sim p(o^A|c^A, \lambda^A). \quad (15)$$

The continuous signal S is generated from the GP-HSMM with parameter Θ^A :

$$S \sim p(S|c^A, \Theta^A). \quad (16)$$

For Agent B, the generative process is the same as that of Agent A. The white nodes in the graphical model represent trainable parameters, which are learned unsupervised from the observations and continuous signals communicated between the two agents.

B. Learning algorithm

Algorithm 1 shows the learning algorithm, and Fig. 5 depicts the overview of the algorithm. First, Agent A generates a continuous signal for n -th observation. Agent B then decides whether to accept or reject the received signal, similar to that using the MHNG algorithm. This procedure is iterated, switching roles for all observation I times. In this section, we explain the details of each computation when Agent A generates a continuous signal and Agent B decides whether to accept or reject it.

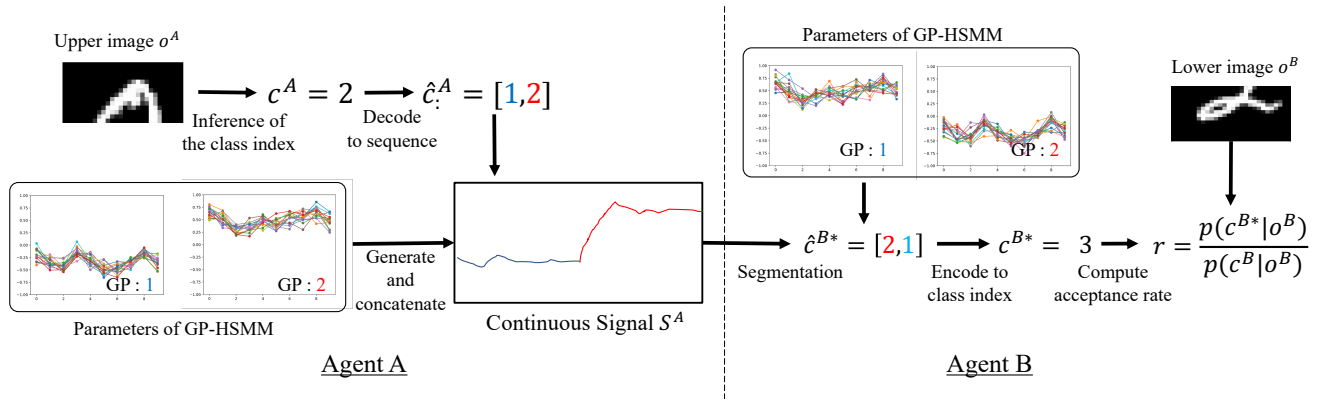


Fig. 5: Overview of the learning algorithm.

Algorithm 1 Learning Algorithm

```

1: Initialize list of  $N$  classes  $C_{list}^A, C_{list}^B$  randomly
2: Initialize list of  $N$  continuous signals  $S_{list}^A, S_{list}^B$  randomly
3:  $\pi^A, \lambda^A, \Theta^A = \text{update\_parameter}(C_{list}^A, S_{list}^A)$ 
4:  $\pi^B, \lambda^B, \Theta^B = \text{update\_parameter}(C_{list}^B, S_{list}^B)$ 
5: for  $i = 1, \dots, I, n = 1, \dots, N$  do
6:   // Agent A generates a continuous signal for  $n$ -th observation
7:    $c^A \sim p(c^A | \lambda^A, \pi^A, o_n^A)$ 
8:    $S^* \leftarrow \text{gen\_singal}(c^A, \Theta^A)$  // Eqs. (18)-(20)
9:
10:  // Agent B decides whether to accept or reject the generated signal
11:   $c^B = C_{list}^B[n]$ 
12:   $c^{B*} \leftarrow \text{recog\_signal}(S^*, \Theta^B)$  // Eqs. (21) and (22)
13:   $r = \min(1, \frac{p(c^{B*}|o_n^B)}{p(c^B|o_n^B)})$ 
14:  if  $r > \text{Unif}(0, 1)$  then
15:     $C_{list}^B[n] = c^{B*}, S_{list}^A[n] = S^*$ 
16:     $\pi^B, \lambda^B, \Theta^B = \text{update\_parameter}(C_{list}^B, S_{list}^B)$ 
17:  end if
18:
19:  // Switch agent roles
20:  // Agent B generates a continuous signal for  $n$ -th observation
21:   $c^B \sim p(c^B | \lambda^B, \pi^B, o_n^B)$ 
22:   $S^* \leftarrow \text{gen\_singal}(c^B, \Theta^B)$  // Eqs. (18)-(20)
23:
24:  // Agent A decides whether to accept or reject the generated signal
25:   $c^A = C_{list}^A[n]$ 
26:   $c^{A*} \leftarrow \text{recog\_signal}(S^*, \Theta^A)$  // Eqs. (21) and (22)
27:   $r = \min(1, \frac{p(c^{A*}|o_n^A)}{p(c^A|o_n^A)})$ 
28:  if  $r > \text{Unif}(0, 1)$  then
29:     $C_{list}^A[n] = c^{A*}, S_{list}^B[n] = S^*$ 
30:     $\pi^A, \lambda^A, \Theta^A = \text{update\_parameter}(C_{list}^A, S_{list}^A)$ 
31:  end if
32: end for

```

TABLE I: Rules of encoding and decoding the classes

class index c	class sequence $\hat{c}$
1	1, 1
2	1, 2
3	2, 1
4	2, 2

1) *Generating continuous signal*: First, Agent A samples a class c^A representing the observation o^A using the current parameter.

$$c^A \sim p(c^A | \lambda^A, \pi^A, o^A) \quad (17)$$

We assume that continuous signals have compositionality and are generated by combining minimum units of signals. Therefore, the class index c^A is decoded to a sequence of L signal classes.

$$\hat{c}_1^A, \hat{c}_2^A, \dots, \hat{c}_L^A \leftarrow \text{decode}(c^A) \quad (18)$$

The continuous signals of each signal class are learned using the GP-HSMM, although the decoding and encoding rules are defined manually. For example, in the experiment, we use the rules listed in Table I with $L = 2$.

A continuous signal S is generated by connecting signals $s_1^A, s_2^A, \dots, s_L^A$ generated by GPs in GP-HSMM.

$$s_l^A \sim \mathcal{GP}(s | \Theta^A, \hat{c}_l^A) \quad : l = 1, \dots, L \quad (19)$$

$$S^* = \text{concatenate}(s_1^A, \dots, s_L^A) \quad (20)$$

The generated S^* is sent to Agent B.

2) *Recognition of continuous signal*: Agent B receives signal S^* and segments it into a signal class sequence $\hat{c}_1^{B*}, \hat{c}_2^{B*}, \dots, \hat{c}_L^{B*}$ by estimating the GP-HSMM parameters. The class index can be obtained by encoding the signal class sequence using the reverse decoding procedure (Eq. (18)).

$$\hat{c}_1^{B*}, \hat{c}_2^{B*}, \dots, \hat{c}_L^{B*} \leftarrow \text{GP-HSMM}(S^* | \Theta^B) \quad (21)$$

$$c^{B*} \leftarrow \text{encode}(\hat{c}_1^{B*}, \hat{c}_2^{B*}, \dots, \hat{c}_L^{B*}) \quad (22)$$

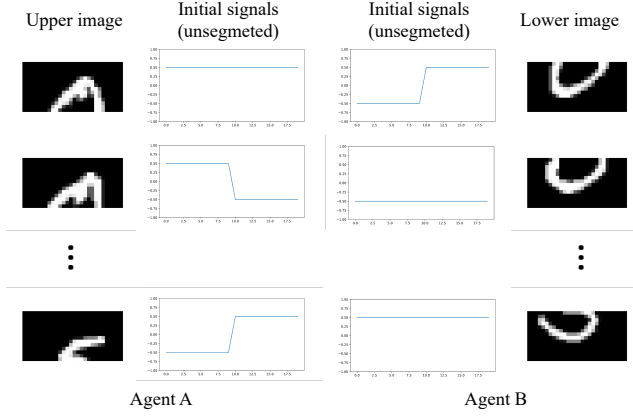


Fig. 6: Examples of assigning initial continuous signals.

TABLE II: Classes estimated by the agents.

Digit of mnist	c^A	c^B
0	4	1
1	3	2
2	2	3
5	1	4

Agent B then decides whether to accept or reject the new class c^{B*} representing observation o^B based on the MHNG algorithm. The acceptance rate can be computed as follows:

$$r = \min\left(1, \frac{p(c^{B*}|o^B)}{p(c^B|o^B)}\right) \quad (23)$$

If $r > \text{uniform}(0, 1)$, class c^{B*} and signal S^* are accepted, and the model parameters are updated using the accepted class and signal.

IV. EXPERIMENT

A. Experimental setup

In the experiments, we used 16 images from the MNIST dataset [17] with labels 0, 1, 2, and 5. Each image was divided into upper and lower halves, and non-negative matrix factorization [18] was applied to compress them into 4-dimensional features. The features of the upper and lower halves of the images were used as observations for Agents A and B, respectively. Four continuous signals, combinations of 0.5 and -0.5, as shown in Fig. 6, were randomly assigned to continuous signals representing the observations as the initial values. We set the number of learning processes $I = 30$ and repeated the experiment ten times with different initial values.

B. Experimental results

First, we discuss the estimated classes, which are the agents' internal states. Classes c^A and c^B were estimated as different class indices for each digit of MNIST in all 10 trials. Table II lists the classes estimated by Agents A and B. Both agents successfully cluster observations through communication using continuous signals. Moreover, the agents need not assign the same value to the same observation because the classes

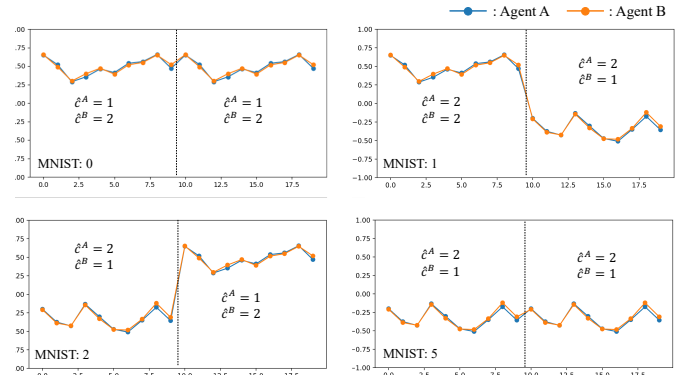


Fig. 7: Estimated continuous signals and signal classes representing MNIST digits. The vertical dotted line represents the boundary of the signal classes.

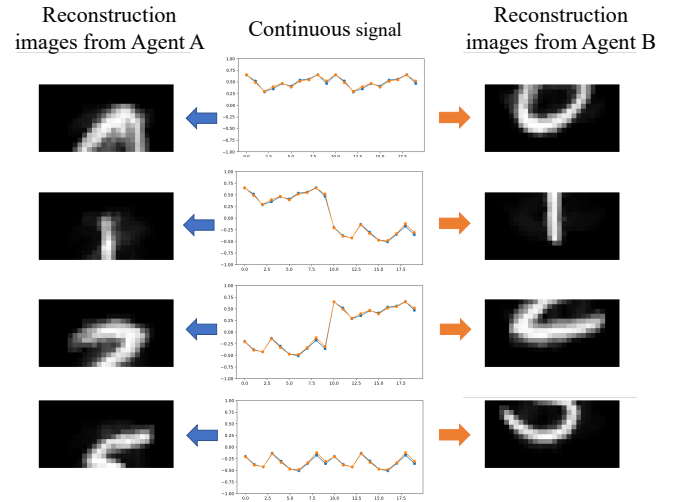


Fig. 8: Reconstruction of images from continuous signals.

are the internal states of each agent; however, it is important to estimate classes that appropriately distinguish observations independently.

Fig. 7 shows the estimated continuous signals representing MNIST digits. Two lines almost overlap, indicating that the two agents share continuous signals representing the same observation, even though the estimated classes, which are internal states, differ. Furthermore, the vertical dotted line represents the boundary of the signal classes, and four continuous signals are appropriately composed using two short signals.

Fig. 8 shows the reconstructed images from four continuous signals. Agents A and B can reconstruct the upper and lower images, respectively, of 0, 1, 2, and 5 from continuous signals. Thus, the meanings of the continuous signals are shared, and symbols that are continuous signals successfully emerge.

V. CONCLUSION

We proposed a novel probabilistic generative model combining MHNG and GP-HSMM to represent continuous signals

as shared symbols through emergent agent communication. The proposed model was evaluated using the upper and lower portions of MNIST as observed images. The results show that the agents can successfully generate and recognize shared continuous signals representing the MNIST digits despite having different internal representations. This study contributes to our understanding of emergent communication between independent individuals and the creation of continuous symbols with compositionality. In the current configuration, communication is established using four signals generated by combinations of two continuous signals. This implies that the number of signals used for communication aligns with the desired number of observable representations. However, as a potential avenue for future research, we contemplate investigating the model's flexibility in estimation with an increase in the number of continuous signal types, considering the segmentation of human phonemes and words.

REFERENCES

- [1] K. Evtimova, A. Drozdov, D. Kiela, and K. Cho, "Emergent communication in a multi-modal, multi-step referential game," *arXiv preprint arXiv:1705.10369*, 2017.
- [2] R. Chaabouni, "Emergent communication at scale," in *International Conference on Learning Representations*, 2022.
- [3] A. Lazaridou and M. Baroni, "Emergent multi-agent communication in the deep learning era," 2020.
- [4] J. Jiang and Z. Lu, "Learning attentional communication for multi-agent cooperation," 2018.
- [5] R. Lowe, Y. Wu, A. Tamar, J. Harb, P. Abbeel, and I. Mordatch, "Multi-agent actor-critic for mixed cooperative-competitive environments," *Conference on Neural Information Processing Systems*, vol. 30, 2017.
- [6] L. Steels, M. Loetzsch *et al.*, "The grounded naming game," *Experiments in cultural language evolution*, vol. 3, pp. 41–59, 2012.
- [7] B. Skyrms, *Signals: Evolution, Learning, and Information*. Oxford, GB: Oxford University Press, 2010.
- [8] D. K. Lewis, *Convention: A Philosophical Study*. Cambridge, MA, USA: Wiley-Blackwell, 1969.
- [9] S. Sukhbaatar, A. Szlam, and R. Fergus, "Learning multiagent communication with backpropagation," *Conference on Neural Information Processing Systems*, vol. 29, 2016.
- [10] J. N. Foerster, Y. M. Assael, N. de Freitas, and S. Whiteson, "Learning to communicate with deep multi-agent reinforcement learning," 2016.
- [11] T. Taniguchi, "Collective predictive coding hypothesis: Symbol emergence as decentralized bayesian inference," *PsyArXiv preprint psyarxiv.com/d2ty6*, 2023.
- [12] T. Taniguchi, Y. Yoshida, Y. Matsui, N. L. Hoang, A. Taniguchi, and Y. Hagiwara, "Emergent communication through metropolis-hastings naming game with deep generative models," *Advanced Robotics*, vol. 37, no. 19, pp. 1266–1282, 2023.
- [13] Y. Hagiwara, K. Furukawa, A. Taniguchi, and T. Taniguchi, "Multiagent multimodal categorization for symbol emergence: emergent communication via interpersonal cross-modal inference," *Advanced Robotics*, vol. 36, no. 5-6, pp. 239–260, 2022.
- [14] R. Okumura, T. Taniguchi, Y. Hagiwara, and A. Taniguchi, "Metropolis-hastings algorithm in joint-attention naming game: Experimental semiotics study," *Frontiers in Artificial Intelligence*, vol. 6, 2023.
- [15] T. Nakamura, T. Nagai, D. Mochihashi, I. Kobayashi, H. Asoh, and M. Kaneko, "Segmenting continuous motions with hidden semi-markov models and gaussian processes," *Frontiers in neurorobotics*, vol. 11, p. 67, 2017.
- [16] M. Nagano and T. Nakamura, "Unsupervised phoneme and word acquisition from continuous speech based on a hierarchical probabilistic generative model," *Advanced Robotics*, vol. 37, no. 19, pp. 1253–1265, 2023. [Online]. Available: <https://doi.org/10.1080/01691864.2023.2252048>
- [17] L. Deng, "The mnist database of handwritten digit images for machine learning research," *IEEE Signal Processing Magazine*, vol. 29, no. 6, pp. 141–142, 2012.
- [18] D. Lee and H. S. Seung, "Algorithms for non-negative matrix factorization," in *Advances in Neural Information Processing Systems*, T. Leen, T. Dietterich, and V. Tresp, Eds., vol. 13. MIT Press, 2000. [Online]. Available: https://proceedings.neurips.cc/paper_files/paper/2000/file/f9d1152547c0bde01830b7e8bd60024c-Paper.pdf